GRADE 8
RESOURCE GUIDE
PART 1

MATHLINKS: GRADE 8
RESOURCE GUIDE: PART 1

THE STANDARDS FOR MATHEMATICAL PRACTICE

In addition to the mathematical topics you will learn about in this course, your teacher will help you become better at what are called the Mathematical Practices. The Standards for Mathematical Practice describe a variety of processes and strategies to help you to be more mathematically proficient and fluent students.

One way to think about the practices is in groupings.

HABITS OF MIND

MP1 Make sense of problems and persevere in solving them
MP6 Attend to precision

REASONING AND EXPLAINING

MP2 Reason abstractly and quantitatively
MP3 Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

MP4 Model with mathematics
MP5 Use appropriate tools strategically

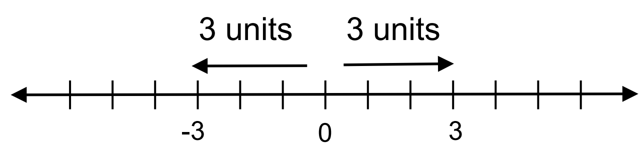
SEEING STRUCTURE AND GENERALIZING

MP7 Look for and make use of structure
MP8 Look for and make use of repeated reasoning

WORD BANK

<table>
<tr><th>word or phrase</th><th>definition</th></tr>
<tr><td>absolute value</td><td>The <u>absolute value</u> $\left| x \right|$ |f a number x is the distance from x to 0 on the number line.

Example: $\left|3\right| = 3$ and $\left|-3\right| = 3$, because both 3 and -3 are 3 units from 0 on the number line.

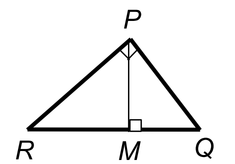
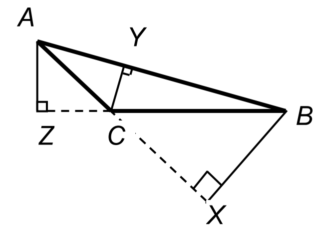
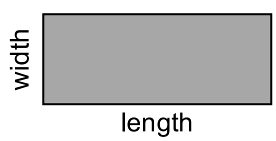

</td></tr>
<tr><td>addend</td><td>In an addition problem, an <u>addend</u> is a number to be added. See <u>sum</u>.

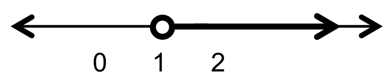
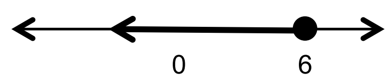
Example: 7 + 5 = 12
addend addend sum</td></tr>
<tr><td>addition property of equality</td><td>The <u>addition property of equality</u> states that if $a = b$ and $c = d$, then $a + c = b + d$. In other words, equals added to equals are equal.

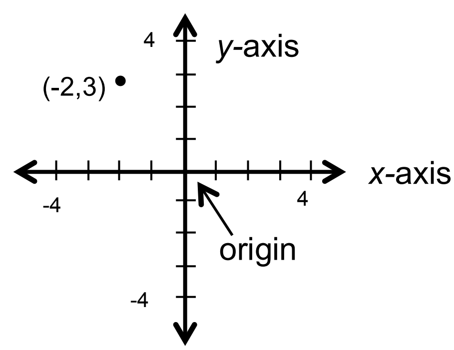
Example: From $y – 2 = 3x + 7$, it follows that $y = 3x + 9$.</td></tr>
<tr><td>additive identity property</td><td>The <u>additive identity property</u> states that $a + 0 = 0 + a = a$ for any number a. In other words, the sum of a number and 0 is the number. We say that 0 is an <u>additive identity</u>. The additive identity property is sometimes called the <u>addition property of zero</u>.

Examples: $3 + 0 = 3$, $0 + 7 = 7$, $-5 + 0 = -5 = 0 + (-5)$</td></tr>
<tr><td>additive inverse property</td><td>The <u>additive inverse property</u> states that $a + (-a) = 0$ for any number a. In other words, the sum of a number and its opposite is 0.
The number $-a$ is the <u>additive inverse</u> of a.

Examples: $3 + (-3) = 0$, $-25 + 25 = 0$</td></tr>
</table>

altitude of a triangle	An altitude of a triangle is a line segment connecting one vertex to a point on the line through the other two vertices, meeting the line at a right angle. The height of the triangle is the length of the altitude. A triangle has three altitudes, each corresponding to a height. Example: In $\triangle PQR$: $\overline{PM}$ is the altitude on $\overline{RQ}$. $\overline{RP}$ is the altitude on $\overline{PQ}$. $\overline{PQ}$ is the altitude on $\overline{RP}$.  Example: In $\triangle ABC$: $\overline{CY}$ is the altitude on $\overline{AB}$. $\overline{AZ}$ is the altitude on $\overline{BC}$. $\overline{BX}$ is the altitude on $\overline{AC}$. 
area	The area of a two-dimensional figure is a measure of *the* size of the figure, expressed in square units. The area of a rectangle is the product of its length and its width.  Area = (length) • (width) Example: If a rectangle has a length of 12 inches and a width of 5 inches, its area is (5)(12) = 60 square inches.
association	In statistics, an association between two variables is a relationship between the variables, so that the variables are statistically dependent. In the case of numerical variables, if the relationship is linear, we refer to a linear association between the variables.
associative property of addition	The associative property of addition states that $a + (b + c) = (a + b) + c$ for any three numbers a, b, and c. In other words, the sum does not depend on the grouping of the addends. Example: 9 + (1 + 14) = (9 + 1) + 14 = 24

associative property of multiplication	The <u>associative property of multiplication</u> states that $(a \bullet b) \bullet c = a \bullet (b \bullet c)$ for any three numbers *a, b,* and *c*. In other words, the product does not depend on the grouping of the factors. Example: $(3 \bullet 4) \bullet 5 = 3 \bullet (4 \bullet 5) = 3 \bullet 4 \bullet 5$
boundary point of a solution set	A <u>boundary point of a solution set</u> is a point for which any segment surrounding it on the number line contains both solutions and non-solutions. If the solution set is an interval, the boundary points of the solution set are the endpoints of the interval. Example: The boundary point for $2 + x > 3$ (or $x > 1$) is $x = 1$. In this case the boundary point is NOT part of the solution set.  Example: The boundary point for $x \leq 6$ is 6. In this case the boundary point IS part of the solution set. 
coefficient	A <u>coefficient</u> is a number or constant factor in a term of an algebraic expression. Example: In the expression $3x + 5$, 3 is the coefficient of the linear term $3x$, and 5 is the constant coefficient.
commutative property of addition	The <u>commutative property of addition</u> states that $a + b = b + a$ for any two numbers a and b. In other words, changing the order of the addends does not change the sum. Example: $14 + 6 = 6 + 14$
commutative property of multiplication	The <u>commutative property of multiplication</u> states that $a \bullet b = b \bullet a$ for any two numbers a and b. In other words, the product does not depend on the order of the factors. Example: $3 \bullet 5 = 5 \bullet 3$

conjecture	A conjecture is a statement that is proposed to be true, but has not been proven to be true nor to be false. Example: After creating a table of sums of odd numbers such as 1 + 3 = 4, 1+ 5 = 6, 5 + 7 = 12, 3 + 9 = 12, etc., we may make a conjecture that the sum of any two odd numbers is an even number. This conjecture can be proven to be true.
consistent system	A system of equations is consistent if it has a solution.
constant term	A constant term in an algebraic expression is a term that has a fixed numerical value. Example: In the expression $5 + 12x^2 - 7$, 5 and -7 are constant terms.
coordinate plane	A coordinate plane is a (Euclidean) plane with two perpendicular number lines (coordinate axes) meeting at a point (the origin). Each point *P* of the coordinate plane corresponds to an ordered pair (*a*,*b*) of numbers, called the coordinates of *P*, determined by where the lines through *P* parallel to each coordinate axis meet the other coordinate axis. Example: The coordinate axes are often referred to as the *x*-axis and the *y*-axis respectively. Points on the *x*-axis have coordinates (*a*, 0), and points on the *y*-axis have coordinates (0, *b*). The origin has coordinates (0, 0). 
deductive reasoning	Deductive reasoning is a form of reasoning in which the conclusion is justified by an argument based on definitions, known facts, and accepted rules of logic. Example: A mathematical proof is a form of deductive reasoning.

denominator	The denominator of the fraction $\frac{a}{b}$ is b. Example: The denominator of $\frac{3}{7}$ is 7.
dependent variable	A dependent variable is a variable whose value is determined by the values of the independent variables. Example: For the function $y = 3x^2 + 1$, y is the dependent variable and x is the independent variable. When a value is assigned to x, the value of y is completely determined.
difference	In a subtraction problem, the difference is the result of subtraction. The minuend is the number from which another number is being subtracted, and the subtrahend is the number that is being subtracted. Example: $\underset{\text{minuend}}{12} - \underset{\text{subtrahend}}{4} = \underset{\text{difference}}{8}$
direct proportion	Two variables are in direct proportion (or direct variation) if one is a multiple of the other. In this case, their ratio is constant, and we say that the variables are in a proportional relationship, or that one varies directly with the other. If the variable y varies directly with the variable x, then $y = kx$, where k is the constant of proportionality. The graph of all points describing this relationship is a straight line through the origin with slope k. Example: If $u = 3t$, then u varies directly with t, that is, u is in direct proportion with t. The constant of proportionality is 3.
direct variation	Direct variation is a relationship where one variable is a multiple of the other. See direct proportion. Example: If $d = 3t$, then d varies directly with t.
distance	The distance between two points on a number line is the absolute value of their difference. Example: The distance from 3 to -4 is $\lvert 3-(-4) \rvert = \lvert 3+4 \rvert = 7$.

distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. Examples: $3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$
division	Division is the mathematical operation that is inverse to multiplication. For $b \neq 0$, division by b is multiplication by the multiplicative inverse $\frac{1}{b}$ of b, $a \div b = a \bullet \left(\frac{1}{b}\right)$. In this division problem, the number a to be divided is the dividend, the number b by which a is divided is the divisor, and the result $a \div b$ of the division is the quotient: $\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$, $\text{divisor}\overline{\smash{)}\text{dividend}}$ with quotient above Other notations for $a \div b$ are $\frac{a}{b}$, a/b, and ab^{-1}.
empirical evidence	Empirical evidence is evidence that is based on experience or observation. Data collected from experimentation are called empirical data.
equality	An equality is a mathematical statement that asserts that two numbers or expressions are equal. Example: $5 + 3 = 4 + 4$ and $x + 1 = 7$ are equality statements.
equal sign	The equal sign is the mathematical symbol " = ". It is placed between expressions to assert that they are equal. Example: In "6 + 4 = 9 + 1," 6 + 4 and 9 + 1 have the same value. Example: In "Solve $5 = x + 1$," one is directed to find x for which equality holds.

equation	An equation is a mathematical statement that asserts the equality of two expressions. When the expressions involve variables, a solution to the equation consists of values for the variables which, when substituted, make the equation true. To solve an equation means to find all values of the variables that make the equation true. Example: $6 + 5 = 14 - 3$ is an equation that involves only numbers. Example: $10 + x = 18$ is an equation that involves numbers and a variable.
equivalent fractions	The fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if they represent the same point on the number line. This occurs if the results of the division problems $a \div b$ and $c \div d$ are equal. Example: Since $\frac{1}{2} = 1 \div 2 = 0.5$ and $\frac{3}{6} = 3 \div 6 = 0.5$, the fractions $\frac{1}{2}$ and $\frac{3}{6}$ are equivalent. Pictorially:  $\frac{1}{2}$ $\frac{3}{6}$
estimate	An estimate is an educated guess.
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. Example: To evaluate the expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$. Example: To evaluate the expression $u^2 - v$ when $u = 5$ and $v = 6$, we calculate $u^2 - v = (5)^2 - 6 = 25 - 6 = 19$.
even number	A number is even if it is divisible by 2. Example: The even integers are 0, ±2, ±4, ±6, … . Example: The even whole numbers are 0, 2, 4, 6, … .

<table>
<tr><td>explicit rule</td><td>An explicit rule (or input-output rule) for a sequence of values is a rule that establishes explicitly an output value for each given input value.

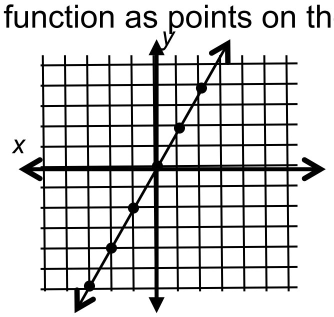
<table>
<tr><th>input value (x)</th><th>output value (y)</th></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>10</td></tr>
<tr><td>3</td><td>16</td></tr>
<tr><td>4</td><td>22</td></tr>
<tr><td>5</td><td>28</td></tr>
<tr><td>6</td><td>34</td></tr>
<tr><td>x</td><td>$6x – 2$</td></tr>
</table>

Example: In the table above, the explicit rule $y = 6x – 2$ give the output value for any input value x. In other words, to get the output value, multiply the input value by 6 and subtract 2.</td></tr>
<tr><td>expression</td><td>A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number.

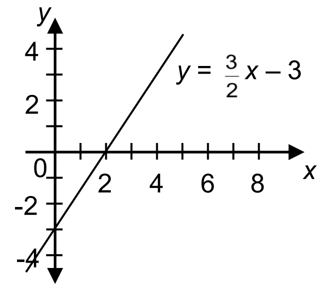
Examples: Some mathematical expressions are $7x$, $a + b$, $4v – w$, and 19.</td></tr>
<tr><td>factor</td><td>In a multiplication problem, a factor is a number or expression being multiplied. See product.

Example: $\underset{\text{factor}}{3} \bullet \underset{\text{factor}}{5} = \underset{\text{product}}{15}$</td></tr>
<tr><td>fourfold way</td><td>The fourfold way refers to a collection of four ways to approach a math problem:
• Numbers (numerical approach, as by a making a t-table)
• Pictures (visual approach, as with a picture or graph)
• Words (verbalizing a solution, orally or in writing)
• Symbols (approaching the problem using algebraic symbols)

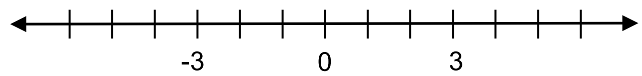
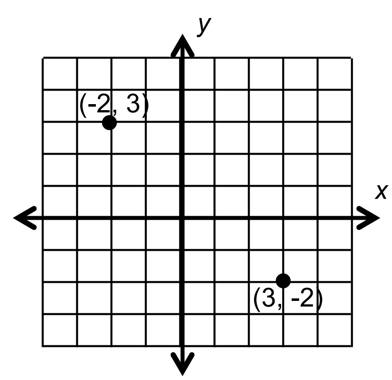
Each approach may lead to a valid solution. Collectively they should lead to a complete and comprehensive solution, one that is readily accessible to more learners and that provides more insight.</td></tr>
</table>

function	A <u>function</u> is a rule that assigns to each input value exactly one output value. A function may also be referred to as a <u>transformation</u> or <u>mapping</u>. The collection of output values is the <u>image</u> of the function. Example: Consider the function $y = 3x + 6$. For any input value, say $x = 10$, there is a unique output value, in this case $y = 36$. This output value is obtained by substituting the value of x into the equation. Example: The function $y = x^2 + 1$ assigns to the input value $x = 2$ the output value $y = 2^2 + 1 = 5$.
generalization	<u>Generalization</u> is the process of formulating general concepts by abstracting common properties from specific cases. Example: We may notice that the sum of two numbers is the same regardless of order (as in 3 + 5 = 5 + 3). We generalize by writing the statement: $a + b = b + a$ for all numbers a and b.
graph of a function	The <u>graph of a function</u> is the set of ordered pairs, each consisting of an input and its corresponding output. If the inputs and outputs are real numbers, then we can represent the graph of a function as points on the coordinate plane. Example: The graph of $y = 2x + 1$ on the coordinate plane 
improper fraction	An <u>improper fraction</u> is a fraction of the form $\frac{m}{n}$ where $m \geq n$ and $n > 0$. Example: The fractions $\frac{3}{2}$, $\frac{17}{4}$, $\frac{9}{9}$ and $\frac{32}{16}$ are improper fractions.
inconsistent system of equations	A system of equations is <u>inconsistent</u> if it has no solutions.

independent variable	An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables. Example: For the function $y = 3x^2 + 1$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.
inductive reasoning	Inductive reasoning is a form of reasoning in which the conclusion is supported by the evidence but is not proved. Example: After creating a table of sums of odd numbers $1 + 3 = 4$, $1 + 3 + 5 = 9$, $1 + 3 + 5 + 7 = 16$, etc., we may reason inductively that the sum of the first n odd numbers is n^2.
inequality	An inequality is a mathematical statement that asserts the relative size or order of two objects. When the expressions involve variables, a solution to the inequality consists of values for the variables which, when substituted, make the inequality true. Example: $5 > 3$ is an inequality. Example: $x + 3 > 4$ is an inequality. Its solution (which is also an inequality) is $x > 1$.
input-output rule	An input-output rule (or explicit rule) is a rule that establishes explicitly an output value for each given input value. See explicit rule, function.
integers	The integers are the whole numbers and their opposites. They are the numbers $0, \pm1, \pm2, \pm3, \ldots$.
inverse operation	The inverse operation to a mathematical operation reverses the effect of the operation. Example: Addition and subtraction are inverse operations. Example: Multiplication and division are inverse operations.
like terms	Terms of a mathematical expression that have the same variable part are referred to as like terms. Example: In the mathematical expression $2x + 6 + 3x + 5$, the terms $2x$ and $3x$ are like terms, and the terms 6 and 5 are like terms.

linear function	A linear function (in variables x and y) is a function that can be expressed in the form $y = mx + b$. The graph of $y = mx + b$ is a straight line with slope m and y-intercept b. Example: The graph of the linear function $y = \frac{3}{2}x - 3$ is a straight line with slope $m = \frac{3}{2}$ and y-intercept $b = -3$. 
linear inequality	A linear inequality in the variables x and y is a mathematical statement which can be written in the form $ax + by + c > 0$.
minuend	A minuend is a number from which another is subtracted. See difference. Example: In 12 – 4 = 8, the minuend is 12.
mixed number	A mixed number is an expression of the form $n\frac{p}{q}$, which is a shorthand for $n + \frac{p}{q}$. Example: The mixed number $4\frac{1}{4}$ (“four and one fourth”) is a shorthand way of writing $4 + \frac{1}{4}$. This should not be confused with the product $4 \bullet \frac{1}{4} = 1$.
multiple	A multiple of a number m is a number of the form $k \bullet m$ for any integer k. Example: The numbers 5, 10, 15, and 20 are multiples of 5, because $1 \bullet 5 = 5$, $2 \bullet 5 = 10$, $3 \bullet 5 = 15$, and $4 \bullet 5 = 20$.
multiplication property of equality	The multiplication property of equality states that if $a = b$ and $c = d$, then $ac = bd$. In other words, equals multiplied by equals are equal. Example: Since $4 + 2 = 5 + 1$ and $10 = 2 \bullet 5$ so $10 \bullet (4 + 2) = 2 \bullet 5 \bullet (5 + 1)$ check $60 = 60$

multiplicative identity property	The multiplicative identity property states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a multiplicative identity. The multiplicative identity property is sometimes called the multiplication property of 1. Example: $4 \bullet 1 = 1 \bullet (4) = 4$.
multiplicative inverse	For $b \neq 0$, the multiplicative inverse of b is the number, denoted by $\frac{1}{b}$, that satisfies $b \bullet \frac{1}{b} = 1$. The multiplicative inverse of b is also referred to as the reciprocal of b. Example: The multiplicative inverse of 4 is $\frac{1}{4}$, since $4 \bullet \frac{1}{4} = 1$.
natural numbers	The natural numbers are the numbers 1, 2, 3, … . Natural numbers are also referred to as counting numbers.
negative number	A negative number is a number that is less than zero. A negative integer is an integer that is less than zero. Example: Some negative numbers are $-\frac{2}{3}$, -8, -54. Example: Some negative integers are -2, -6, -54. **Heads Up!** Zero is neither negative nor positive.
numerator	The numerator of the fraction $\frac{a}{b}$ is a. Example: The numerator of $\frac{3}{7}$ is 3.
numerical coefficient	The numerical coefficient of a term in an algebraic expression is the numerical factor of the term. See coefficient. Example: The numerical coefficient of $6x^2$ is 6. Example: The numerical coefficient of $-x$ is -1. Example: The numerical coefficient of $3(x + 2)$ is 3.
odd number	A number is odd if it is *not* divisible by 2. Example: The odd integers are ± 1, ± 3, ± 5, … .

opposite of a number	The opposite of a number *n*, written $-n$, is its additive inverse. Algebraically, the sum of a number and its opposite is zero. Geometrically, the opposite of a number is its reflection through zero on the number line. Example: The opposite of 3 is -3, because 3 + (-3) = -3 + 3 = 0. Similarly, the opposite of -3 is -(-3) = 3. Thus 3 and -3 are opposites of one another.  **Heads Up!** The opposite of a number does NOT have to be negative. Also, the opposite of a number is not necessarily its absolute value.
order of operations	An order of operations is a convention, or set of rules, that specifies in what order to perform the operations in an algebraic expression. The standard order of operations is as follows: 1. Do the operations in parentheses first. (Use rules 2-4 inside the parentheses.) 2. Calculate all the expressions with exponents. 3. Multiply and divide in order from left to right. 4. Add and subtract in order from left to right. In particular, multiplications and divisions are carried out before additions and subtractions. Example: $\frac{3^2+(6\bullet 2-1)}{5}=\frac{3^2+(12-1)}{5}=\frac{3^2+(11)}{5}=\frac{9+(11)}{5}=\frac{20}{5}=4$
ordered pair	An ordered pair of numbers is a pair of numbers with a specified order. Ordered pairs are denoted (a, b), (x, y), etc. Example: Ordered pairs of numbers are used to represent points in a coordinate plane. The ordered pair (3, -2) represents the point with *x*-coordinate 3 and *y*-coordinate -2. This is different from the ordered pair (-2, 3). 

<table>
<tr>
<td>perimeter</td>
<td>The <u>perimeter</u> of a plane figure is the length of the boundary of the figure.

Example: The perimeter of a square is four times its side-length.

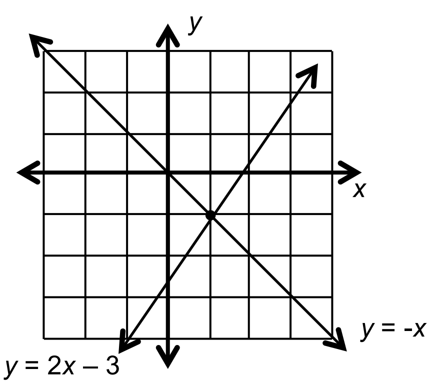
Example: The perimeter of a rectangle is twice the length plus twice the width.

Example: The perimeter of a circular disk is its circumference, which is π times its diameter.</td>
</tr>
<tr>
<td>place value number system</td>
<td>A <u>place value number system</u> is a positional number system in which the value of a digit in a number is determined by its location or place.

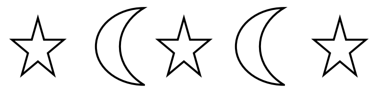
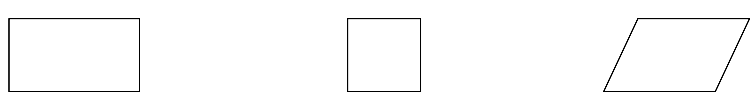
Example: In the number 7,863.21, the 8 is in the hundreds place and represents 800. The 1 is in the hundredths place and represents 0.01.

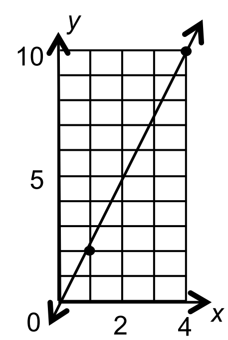
<table>
<tr><td>ten millions</td><td>millions</td><td>hundred thousands</td><td>ten thousands</td><td>thousands</td><td>hundreds</td><td>tens</td><td>ones</td><td>•</td><td>tenths</td><td>hundredths</td><td>thousandths</td><td>ten thousandths</td></tr>
</table>
</td>
</tr>
<tr>
<td>point of intersection</td>
<td>A <u>point of intersection</u> of two curves is a point where the curves meet.

Example: The two straight lines in the plane with equations $y = -x$ and $y = 2x - 3$ have point of intersection (1, -1).


</td>
</tr>
</table>

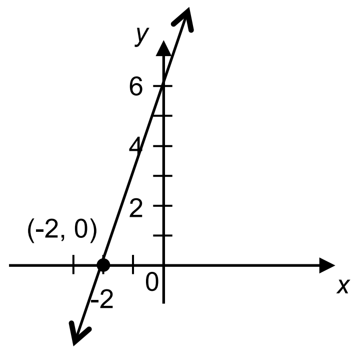
product	A product is the result of multiplying two or more numbers or expressions. The numbers or expressions being multiplied to form the product are factors of the product. Example: The product of 7 and 8 is 56, written $7 \bullet 8 = 56$. The numbers 7 and 8 are both factors of 56.
proof	A proof of a mathematical statement is an argument based on definitions, previously established theorems, and accepted rules of logic to justify the statement. See deductive reasoning.
proper fraction	A proper fraction is a fraction of the form $\frac{m}{n}$, where m and n are natural numbers and $m < n$. Example: The fractions $\frac{1}{2}$ and $\frac{5}{6}$ are proper fractions.
proportion	A proportion is a statement that two ratios are equal. Example: The equation $\frac{3}{25} = \frac{12}{100}$ is a proportion. It can also be written $3 : 25 = 12 : 100$.
proportional	Two quantities are proportional if one is a multiple of the other. We say that y is proportional to x if $y = kx$, where k is the constant of proportionality. Example: In a scale drawing of a figure, lengths associated with the scale drawing are proportional to the corresponding lengths associated with the original figure. The constant of proportionality is the scale factor of the scale drawing.
quotient	In a division problem, the quotient is the result of the division. See division. Example: In $12 \div 3 = 4$, the quotient is 4.
rate	A rate is a ratio in which the numbers have units attached to them. Example: $\frac{\text{20 miles}}{\text{15 minutes}}$ is a rate.

ratio	A <u>ratio</u> is a comparison of two numbers by division. The ratio of a to b is denoted by $a : b$ (read " a to b "), or by $\frac{a}{b}$. Example: The ratio of stars to moons in this picture is 3 to 2, or 1.5 stars for each moon.  The ratio of 3 to 2 may be denoted by 3:2, or by $\frac{3}{2} = 1.5$.
rational numbers	<u>Rational numbers</u> are quotients of integers. Rational numbers can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Example: $\frac{3}{5}$ is rational because it *is* a quotient of integers. Example: $2\frac{1}{3}$ and 0.7 are rational numbers because they *can be* expressed as quotients of integers ($2\frac{1}{3} = \frac{7}{3}$ and $0.7 = \frac{7}{10}$). Example: $\sqrt{2}$ and π are NOT rational numbers. They *cannot be* expressed as a quotient of integers.
reciprocal	The <u>reciprocal</u> of a nonzero number is its multiplicative inverse. See <u>multiplicative inverse</u>. Example: The reciprocal of 3 is $\frac{1}{3}$. The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.
rectangle	A <u>rectangle</u> is a quadrilateral with four right angles. In a rectangle, opposite sides are parallel and have equal length. Example: A square is a rectangle with four congruent sides.  rectangle square not a rectangle

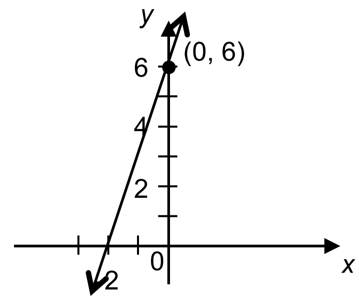
recursive rule	A <u>recursive rule</u> for a sequence of values is a rule that specifies each value based on previous values in the sequence. In order to determine a sequence by a recursive rule, an appropriate number of starting values for the sequence must be identified. Example: The recursive rule “start with 4 and add 6 each time” determines the sequence 4, 10, 16, 22, 28, 34…
reflexive property of equality	The <u>reflexive property of equality</u> states that $a = a$ for any number a. Example: 7 = 7
rounding	<u>Rounding</u> refers to replacing a number by a nearby number that is easier to work with or that better reflects the precision of the data. Typically, rounding requires the changing of digits to zeros. Example: 15,632 rounded to the nearest thousand is 16,000. Example: 12.83 rounded to the nearest tenth is 12.80, or 12.8.
simplify	<u>Simplify</u> refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. Example: $2x + 6 + 5x + 3 = 7x + 9$ Example: $\frac{8}{12} = \frac{2}{3}$
slope of a line	The <u>slope of a line</u> is the vertical change (change in the y-value) per unit of horizontal change (change in the x-value). The slope is sometimes described as the ratio of the “rise to the run.” Example: The slope of the line through (1, 2) and (4, 10) is $\frac{8}{3}$: $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{(\text{difference in y})}{(\text{difference in x})} = \frac{(10 - 2)}{(4 - 1)} = \frac{8}{3}$ 
slope-intercept form	The <u>slope-intercept form</u> of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. Example: The equation $y = 2x + 3$ determines a line with slope 2 and y-intercept 3.

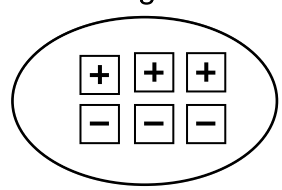
solution set	A solution set is the set of values that satisfy a given set of equations or inequalities.
solution to an equation	A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true. Example: The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	Solve an equation refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. Example: To solve the equation $2x = 6$, one might think “two times what number is equal to 6?” The only value for x that satisfies this condition is 3, which is then the solution.
substitution	Substitution refers to replacing a value or quantity with an equivalent value or quantity. Example: If $x + y = 10$, and we know that $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.
subtrahend	In a subtraction problem, the subtrahend is the number that is being subtracted from another. See difference. Example: In $12 – 4 = 8$, the subtrahend is 4.
sum	A sum is the result of addition. In an addition problem, the numbers to be added are addends. Example: $\underset{\text{addend}}{7} + \underset{\text{addend}}{5} = \underset{\text{sum}}{12}$ Example: In $3 + 4 + 6 = 13$, the addends are 3, 4, and 6, and the sum is 13.
symmetric property of equality	The symmetric property of equality states that if $a = b$, then $b = a$.

system of linear equations	A system of linear equations is a set of two or more linear equations in the same variables.
term	The terms in a mathematical expression involving addition (or subtraction) are the quantities being added (or subtracted). Terms that have the same variable part are referred to as like terms. Example: Expression: $2x + 6 + 3x + 5$ Terms: $2x$, 6, $3x$, and 5 Like terms: The terms $2x$ and $3x$ are like terms because they have the same variable part (x). The terms 6 and 5 are like terms because they are both constants (or their variable part can be considered as x^0).
transitive property of equality	The transitive property of equality states that if $a = b$ and if $b = c$, then $a = c$.
unit fraction	A unit fraction is a fraction of the form $\frac{1}{m}$, where m is a natural number. Example: The unit fractions are $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$.
unit price	A unit price is a price for one unit of measure. Example: \$1.25 per pound, which can be written $\frac{1.25 \text{ dollars}}{1 \text{ pound}}$ or $1.25 \frac{\text{dollars}}{\text{pounds}}$.
unit rate	A unit rate is a rate for one unit of measure. Example: 80 miles per hour, which can be written $\frac{80 \text{ miles}}{1 \text{ hour}}$ or $80 \frac{\text{miles}}{\text{hour}}$.

variable	A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to functions, to quantities that vary in a relationship, or to unknown quantities in equations and inequalities. Example: In the equation $d = r\,t$, the quantities d, r, and t are variables. Example: In the expression $2x + 6 + 4x$, the variable x may be referred to as the unknown.
vertical line test	The vertical line test asserts that a subset of the plane is the graph of a function if and only if each vertical line in the plane meets the set in at most one point. In particular, if a vertical line intersects a set in the plane in more than one point, the set cannot be the graph of a function.
whole numbers	The whole numbers are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, ….
x-intercept	The x-intercept of a line is the x-coordinate of the point at which the line crosses the x-axis. It is the value of x that corresponds to $y = 0$. Example: The x-intercept of the line $y = 3x + 6$ is -2. If $y = 0$, then $x = -2$. 

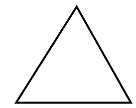
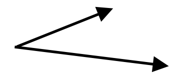
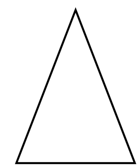
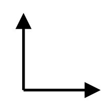
<table>
<tr>
<td>y-intercept</td>
<td>The <u>y-intercept</u> of a line is the y-coordinate of the point at which the line crosses the y-axis. It is the value of y that corresponds to $x = 0$.

Example: The y-intercept of the line $y = 3x + 6$ is 6.
If $x = 0$, then $y = 6$.

</td>
</tr>
<tr>
<td>zero pair</td>
<td>In the signed counters model, a positive and a negative counter together form a <u>zero pair</u>.

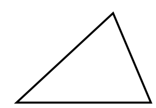
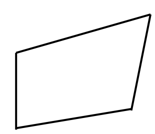
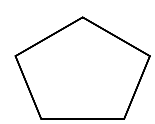
Example: Let $\boxed{+}$ represent a positive counter and
let $\boxed{-}$ represent a negative counter. Then


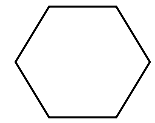
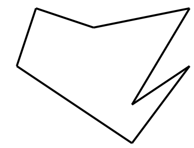
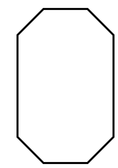
is an example of a collection of (three) zero pairs.</td>
</tr>
</table>

GREEK AND LATIN WORD ROOTS

equi	"equal": An equilateral triangle has three equal sides. The sides of an equilateral polygon have the same length. 
gon	"angle": A pentagon has five angles. 
hedron	"seat," "base"; refers to faces of a solid figure. A decahedron is a polyhedron with ten faces.
iso	"equal," "the same": An isosceles triangle has two equal sides. Isomorphic objects have the same form. Some folks maintain their equilibrium by doing isometric exercises. "equi" comes from Latin, while "iso" comes from Greek. 
lateral	"side": A quadrilateral has four sides. A movement sideways is a lateral movement.
ortho	"right angle," "upright," "correct": Two lines are orthogonal if they are perpendicular. Orthorhombic crystals have three axes that come together at right angles. An orthodontist corrects teeth. An orthodox person seeks to be correct. 
peri	"around," "about": The perimeter of a plane figure is the distance around it. A periscope allows one to see around an obstruction. Paul Erdös was a well-known peripatetic mathematician.
poly	"many": A polygon has many (three or more) angles. A polygraph records several body activities simultaneously. A polygamist has more than one spouse. Los Angeles schools have a polyglot student body.

NUMBER PREFIXES

uni	"one": The units place is to the left of the decimal. We form the union of sets by unifying them. Some problems have unique solutions. A unicycle has one wheel. A unicorn has one horn.
mono	"alone," "one": A monomial is a polynomial with only one term. A monorail has one rail. He delivered his monologue about monogamous relationships in a dull monotone.
bi, bis	"two": The binary number system has base two. We bisect an angle. Property taxes are collected biannually, in March and November. Congressional elections are held biennially, in even-numbered years. Man and ape are bipeds.
di, dy	"two": The angle between two planes is a dihedral angle. A dyadic rational number is the quotient of an integer and a power of two. Physicists deal with dipoles, chemists with dioxides. A diphthong is a gliding speech form with two sounds. "di" comes from Greek, "bi" from Latin.
tri	"three": A triangle has three sides. A triathlon has three events: cycling, swimming, and running. 
quad	"four": A quadrilateral is a polygon with four sides. Coordinate axes divide the plane into four quadrants. The Olympic Games are a quadrennial event. 
tetra, tra	"four": A tetrahedron is a solid figure with four faces. A trapeze is a flying quadrilateral. "tetra" stems from Greek, while "quad" stems from Latin. The "tra" in "trapezoid" and "trapeze" is an abbreviated form of "tetra."
penta	"five": A pentagon has five sides. A pentahedron is a solid figure with five faces. The Pentagon Building in Washington, D.C., has five sides. 

hexa	"six": A hexagon has six sides. The cells in a beehive are hexagonal. 
hepta	"seven": A heptagon has seven sides. A heptahedron has seven faces. 
octa	"eight": An octagon has eight sides. The three coordinate planes divide space into eight octants. The musical octave has eight whole notes. An octopus has eight tentacles. 
nona	"nine": A nonagon has nine sides. A nonagenarian is in his or her nineties. Another (rarely used) word for a nine-sided polygon is "enneagon." The prefix "nona" comes from Latin, "ennea" from Greek.
deca	"ten": A decagon has ten sides. A decade has ten years. A decathlon has ten track-and-field events.
dodeca	"twelve": A dodecahedron has twelve faces. The regular dodecahedron is one of the five Platonic solids. It is occasionally used for calendars.
icosa	"twenty": An icosahedron has twenty faces. The regular icosahedron is one of the five Platonic solids. It has twelve vertices, and it can be obtained from a regular dodecahedron by placing a vertex at the center of each face of the dodecahedron.
cent	"hundred": A centipede has a hundred legs. A centimeter is a hundredth of a meter. The St. Louis fair celebrated the centennial of the Lewis and Clark expedition.
milli	"thousand": We have entered a new millennium. A millisecond is a thousandth of a second.

INTEGER CONCEPTS AND MODELS

Elevation refers to position relative to sea level (in meters or feet).

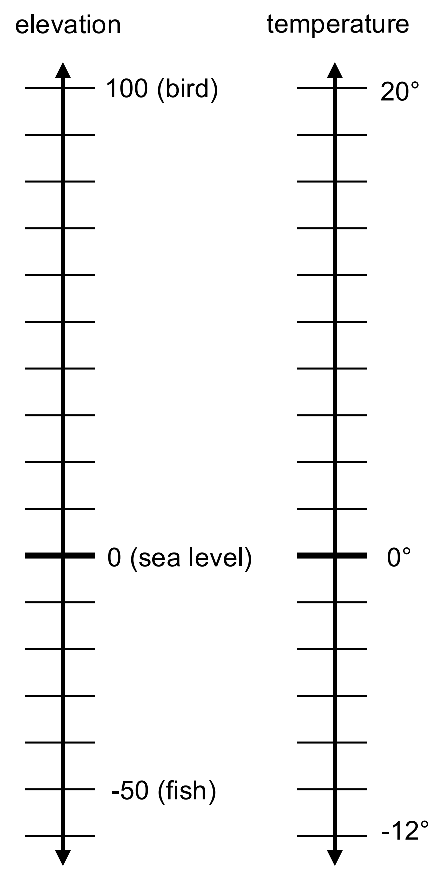
A positive elevation refers to a height above sea level.
A negative elevation refers to a depth below sea level.
Elevation 0 refers to sea level.

Distance is always greater or equal to zero. Distance is never negative.

The absolute value of a number is its distance from zero.

Elevation	**Distance from zero (sea level)**	**Absolute value equation for the distance**
+10 m	10 m	$\lvert 10 \rvert = 10$
-20 m	20 m	$\lvert -20 \rvert = 20$
0	0	$\lvert 0 \rvert = 0$

Example 1: (elevation)	Example 2: (temperature)
Find the distance between a bird flying at 100 m above sea level and a fish swimming directly below it at 50 m below sea level.	At 1:00 P.M. the temperature was 20°. At 6:00 P.M., the temperature was -12°. What was the temperature change?
$\lvert 100 - (-50) \rvert = \lvert 150 \rvert = 150$	$-12 - (20) = -32$
$\lvert (-50) - 100 \rvert = \lvert -150 \rvert = 150$	The temperature change was -32°.



The Counter Model

The counter model is a concrete model that can be used to model computation with integers.

Let $\boxed{+}$ represent a positive counter and $\boxed{-}$ represent a negative counter. A zero pair is a pair with one positive counter and one negative counter.

one zero pair: $\begin{matrix}\boxed{+}\\ \boxed{-}\end{matrix}$ three zero pairs: $\begin{matrix}\boxed{+} & \boxed{+} & \boxed{+}\\ \boxed{-} & \boxed{-} & \boxed{-}\end{matrix}$

Both representations have a value of zero.

Some possible models for the given integer values:

Integer:	0	4	-2
simplest representation:		$\begin{matrix}\boxed{+} & \boxed{+}\\ \boxed{+} & \boxed{+}\end{matrix}$	$\boxed{-}\ \boxed{-}$
another representation:	$\begin{matrix}\boxed{+} & \boxed{+} & \boxed{+} & \boxed{+}\\ \boxed{-} & \boxed{-} & \boxed{-} & \boxed{-}\end{matrix}$	$\begin{matrix}\boxed{+} & \boxed{+} & \boxed{-} & \boxed{-}\\ \boxed{+} & \boxed{+} & \boxed{+} & \boxed{+}\end{matrix}$	$\begin{matrix}\boxed{+} & \boxed{+} & \boxed{+} & & \\ \boxed{-} & \boxed{-} & \boxed{-} & \boxed{-} & \boxed{-}\end{matrix}$

The Temperature Change Model

Suppose scientists discover an amazing way to control the temperature of liquid. They've invented "hot pieces" and "cold nuggets" to change the temperature of the liquid. If you add a hot piece to a liquid, the liquid heats up by one degree. If you add a cold nugget to the liquid, the liquid cools down by one degree. If you have a liquid that you want to cool down, place some cold nuggets in it. They never melt! Too cold now? Put some hot pieces in.

How the temperature change model works:		For 1 piece or 1 nugget:
Hot **P**ieces **P**ositive $(+)$	Put in Heat → Hotter	+ (+1) = 1
	Remove Heat → Colder	- (+1) = -1
Cold **N**uggets **N**egative $(-)$	Put in Cold → Colder	+ (-1) = -1
	Remove Cold → Hotter	- (-1) = +1

INTEGER OPERATIONS

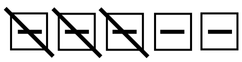
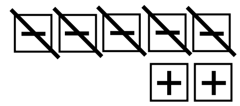
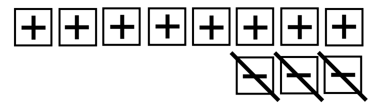
Integer Addition Using Signed Counters		
-3 + (-5) = -8	-3 + 5 = 2	3 + (-5) = -2
$\boxed{-}\ \boxed{-}\ \boxed{-}$ $\boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}$	$\boxed{-}\ \boxed{-}\ \boxed{-}$ $\boxed{+}\ \boxed{+}\ \boxed{+}\ \boxed{+}\ \boxed{+}$	$\boxed{+}\ \boxed{+}\ \boxed{+}$ $\boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}\ \boxed{-}$
• Begin with a value of zero. • Build negative 3. • The (+) means to add. • Add 5 negative counters. • The result is 8 negative counters.	• Begin with a value of zero. • Build negative 3. • The (+) means to add. • Add 5 positive counters. • The result is 2 positive counters.	• Begin with a value of zero. • Build positive 3. • The (+) means to add. • Add 5 negative counters. • The result is 2 negative counters.

- When drawing pictures DO NOT use plus signs for adding, as they will look like the picture of a positive counter (+1). Placing a counter on your workspace is the action of adding.
- Also, DO NOT use equal signs, because the final picture you see in front of you is the result, and the equal sign may just get in the way.

Using a Think Aloud Strategy for Addition of Integers	
Think counter model: (begin with a work space = 0)	Connect to temperature change model: (begin with temperature = zero degrees)
• Build ___________ ___________. positive/negative quantity	• Create a liquid temp. of _____ degree(s). number
• The _________ means to add. operation sign	• The __________ means to put in. operation sign
• Add _____ ____________counter(s). quantity positive/negative	• Put in _____ ___________________. quantity hot piece(s) / cold nugget(s)
• The result is _____ _______counter(s). quantity pos. / neg.	• The liquid is now _____ degree(s). number

Rules for Addition of Two Integers

- When the addends have the same sign, add the absolute values. Use the original sign in the answer.
- When the addends have different signs, subtract the absolute values. Use the sign of the addend with the greater absolute value in the answer.

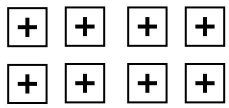
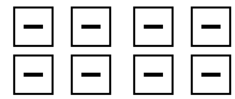
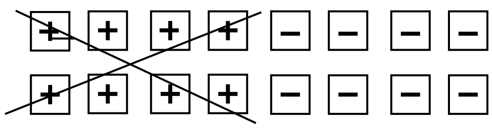
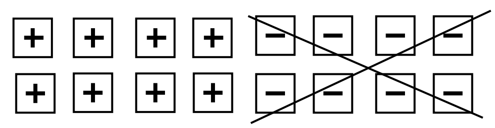
Integer Subtraction Using Signed Counters		
-5 – (-3) = -2  • Begin with a value of zero. • Build negative 5. • The (–) means to <u>subtract</u>. • I do not need zero pairs. • Subtract 3 negative counters. • The result is 2 negative counters.	-3 – (-5) = 2  • Begin with a value of zero. • Build negative 3. • The (–) means to <u>subtract</u>. • I need at least 2 zero pairs. • Subtract 5 negative counters. • The result is 2 positive counters.	5 – (-3) = 8  • Begin with a value of zero. • Build positive 5. • The (–) means to <u>subtract</u>. • I need at least 3 zero pairs. • Subtract 3 negative counters. • The result is 8 positive counters.

- When drawing pictures DO NOT use minus signs for subtracting, as they will look like the picture of a negative counter (-1). Removing a counter from your workspace is the action of subtracting.
- Also, DO NOT use equal signs, because the final picture you see in front of you is the result, and the equal sign may just get in the way, or look like two minus signs.

Using a Think Aloud Strategy for Subtraction of Integers	
<u>Think counter model</u>: (begin with a work space = 0) • Build __________ ___________ . positive/negative quantity • The __________ means to <u>subtract.</u> operation sign • Add zero pairs if needed. • Subtract _____ ____________counter(s). quantity positive/negative • The result is _____ ________counter(s). quantity pos. / neg.	<u>Connect to temperature change model</u>: (begin with temperature = zero degrees) • Create a liquid temperature of _____ degree(s). number • The ___________ means to <u>remove</u>. operation sign • Add zero pairs if needed. • Remove _____ ____________________. quantity hot piece(s) / cold nugget(s) • The liquid is now _____ degree(s). number

Rules for Subtraction of Integers

Rule: In symbols, $a - b = a + (-b)$ and $a - (-b) = a + b$.
In words, subtracting a quantity gives the same result as adding its opposite.

<table>
<tr><th colspan="2">Integer Multiplication Using Signed Counters</th></tr>
<tr><td>2 (4) = 8

• Begin with a value of zero.
• The first factor is (+). We will put 2 groups on the workspace.
• The second factor is positive. Each group will contain 4 positive counters.
• The result is 8 positive counters.</td><td>2 (-4) = -8

• Begin with a value of zero.
• The first factor is (+). We will put 2 groups on the workspace.
• The second factor is negative. Each group will contain 4 negative counters.
• The result is 8 negative counters</td></tr>
<tr><td>-2 (4) = -8

• Begin with a value of zero.
• The first factor is (–). We will remove 2 groups from the workspace.
• The second factor is positive. Each group will contain 4 positive counters.
• I need at least 8 zero pairs to do this.
• The result is 8 negative counters.</td><td>-2(-4) = 8

• Begin with a value of zero.
• The first factor is (–). We will remove 2 groups from the workspace.
• The second factor is negative. Each group will contain 4 negative counters.
• I need at least 8 zero pairs to do this.
• The result is 8 positive counters.</td></tr>
</table>

- When drawing pictures DO NOT use multiplication signs. They are not necessary. Placing or removing groups of counters on your workspace is the action of multiplying.
- Also, DO NOT use equal signs, because the final picture you see in front of you is the result, and the equal sign may just get in the way.

Think Aloud Strategies for Multiplication of Integers

Think Counter Model:

(begin with a workspace = 0)

- The first factor is _______ (positive / negative). We will ______ (place / remove) _____ (quantity) group(s) ________ (in / from) the workspace.
- The second factor is _________ (positive / negative). Each group will contain ______ (quantity) _________ (positive / negative) counter(s).
- I need at least ________ (quantity) zero pairs to do this.
- The result is ________ (quantity) _____________ (positive / negative) counter(s).

Connect to the Temperature Change Model:

(begin with temperature = zero degrees)

- The first factor is __________ (positive / negative). We will __________ (put / remove) _____ (quantity) group(s) ______ (in / from) the liquid.
- The second factor is _________ (positive / negative). Each group will contain ______ (quantity) ________________ (hot piece(s) / cold nugget(s)).
- I need at least _______ (quantity) zero pairs to do this.
- The liquid is now _________ (number) degree(s).

Rules for Multiplication of Integers

Rule 1: The product of two numbers with the same sign is a positive number.
THINK: (+)(+) = (+) and (–)(–) = (+)

Rule 2: The product of two numbers with opposite signs is a negative number.
THINK: (+)(–) = (–) and (–)(+) = (–)

<table>
<tr><th colspan="2">Integer Division</th></tr>
<tr><td colspan="2">Since multiplication and division are inverse operations, the rules for multiplying integers above apply to dividing integers as well.</td></tr>
<tr><th>Multiplication Equation</th><th>Related Division Equations</th></tr>
<tr><td rowspan="2">$4 \bullet 5 = 20$</td><td>$20 \div 4 = 5$ or $\frac{20}{4} = 5$</td></tr>
<tr><td>$20 \div 5 = 4$ or $\frac{20}{5} = 4$</td></tr>
<tr><td rowspan="2">$4 \bullet (-5) = -20$</td><td>$-20 \div 4 = -5$ or $\frac{-20}{4} = -5$</td></tr>
<tr><td>$-20 \div (-5) = 4$ or $\frac{-20}{-5} = 4$</td></tr>
<tr><td rowspan="2">$-4 \bullet (-5) = 20$</td><td>$20 \div (-4) = -5$ or $\frac{20}{-4} = -5$</td></tr>
<tr><td>$20 \div (-5) = -4$ or $\frac{20}{-5} = -4$</td></tr>
<tr><td colspan="2">Rules for Division of Integers

Rule 1: The quotient of two numbers with the same sign is a positive number.

THINK: $\frac{(+)}{(+)} = (+)$ and $\frac{(-)}{(-)} = (+)$

Rule 2: The quotient of two numbers with opposite signs is a negative number.

THINK: $\frac{(+)}{(-)} = (-)$ and $\frac{(-)}{(+)} = (-)$</td></tr>
</table>

- Even though discovering integer division rules using the above reasoning is quick and efficient, trying to use the counter model for dividing integers is an interesting exploration.

Why is Division by Zero Undefined?

In mathematics, division by zero is undefined. This is a consequence of the laws of arithmetic, which guarantee that the number 0 has no multiplicative inverse. There are various ways of understanding why we should not be able to divide by zero.

A numerical example:

$4\overline{)12}^{\,3}$ but what about $0\overline{)12}^{\,\square}$?

Since multiplication and division are inverses of one another, we know that the division fact $12 \div 4 = 3$ is related to the multiplication fact $4 \bullet 3 = 12$.

Now consider $12 \div 0 = \square$. This leads to a multiplication fact $0 \bullet \square = 12$. What can be multiplied by 0 to obtain 12? The answer, of course, is that no such number exists.

A real-life example:

Say there are 12 students that will be placed in groups of 4. Dividing $12 \div 4$ tells us that 3 groups of 4 can be formed.

Now there are 12 students that will be placed in groups of 0. Dividing $12 \div 0$ tells us that $\square$ groups of 0 can be formed. How many groups of 0 make 12? Again, no such number exists.

An explanation using a pattern:

$\frac{10}{1} = 10$, $\frac{10}{0.1} = 100$, $\frac{10}{0.01} = 1{,}000$, $\frac{10}{0.001} = 10{,}000$, etc.

As the denominator gets smaller and smaller, the quotient grows and grows. So $\frac{10}{0}$ is infinite (in some sense). So there is no number that satisfies the equation $\frac{10}{0} = \square$.

The mathematical explanation:

The definition of "division by b" is multiplication by the multiplicative inverse (reciprocal) of b. The number zero has no multiplicative inverse—there is no number a satisfying $0 \bullet a = 1$, because $0 \bullet a = 0$ for all a. Consequently, we cannot divide by zero.

ORDER OF OPERATIONS

Order of Operations

The order in which we perform mathematical calculations is a matter of convention agreed upon by the mathematical community. The order of operations is as follows:

1. Simplify expressions that are grouped (following 2, 3, and 4 below).
2. Perform all operations with exponents and roots.
3. Perform multiplication and division from left to right.
4. Perform addition and subtraction from left to right.

Example 1:	Example 2:	Example 3:	Example 4:
$6 - 5 \bullet 3 = 6 - 15$ $= -9$	$(5-2)^2 = 3^2$ $= 9$	$-5^2 = -25$ think: $0 - 5^2 = 0 - 25$	$12 \div 3 \bullet 2 = 4 \bullet 2$ $= 8$
multiplication before subtraction	parentheses before exponents	exponents before subtraction	multiplication and division from left to right

Caution: Please Be Careful with Aunt Sally

The statement, “Please Excuse My Dear Aunt Sally,” is commonly used to help students remember the correct order of operations because the first letter of each word roughly corresponds to the conventions (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction).

However, caution should be used because the operations are not always performed in that order.

According to the conventions, multiplication and division have equal status in the hierarchy, as do addition and subtraction. Consequently we **do not** necessarily perform all multiplication before division, and we **do not necessarily** perform all addition before subtraction.

Example: $8 \div 4 \bullet 2 = 2 \bullet 2 = 4$, **Right!**
$8 \div 4 \bullet 2 = 8 \div 8 = 1$, **Wrong!**

Note: Generally, expressions such as $8 \div 4 \cdot 2$ are regarded as ambiguous. To avoid ambiguity, include parentheses or fraction notation for clarity.

Example: $8 \div 4 \cdot 2 = (8 \div 4) \cdot 2 = \left(\frac{8}{4}\right)(2)$

Using Order of Operations to Simplify More Complex Expressions		
Order of Operations	**Example:** $\dfrac{\mathbf{2^3 \bullet 3(4 - 2)}}{\mathbf{4 + 2 \bullet 10}}$	**Comments**
1. Simplify expressions within grouping symbols.	$\dfrac{2^3 \bullet 3(2)}{4 + 2 \bullet 10}$	Parentheses are grouping symbols, and $4 - 2 = 2$. **Heads Up!** the fraction bar, used for division, is also a grouping symbol, so the numerator and denominator must be simplified completely prior to dividing.
2. Calculate powers and roots.	$\dfrac{8 \bullet 3(2)}{4 + 2 \bullet 10}$	$2^3 = 2 \bullet 2 \bullet 2 = 8$.
3. Perform multiplication and division from left to right.	$\dfrac{8 \bullet 6}{4 + 20} = \dfrac{48}{4 + 20}$	Perform the multiplications $3(2) = 6$ and $2 \bullet 10 = 20$, and t $8 \bullet 6 = 48$ hen also .
4. Perform addition and subtraction from left to right.	$\dfrac{48}{24} = 2$	Perform the addition $4 + 20 = 24$. Now the groupings in both the numerator and denominator have been simplified, so the final division can be performed.

Using Order of Operations to Simplify More Complex Expressions (continued)

Another example:

1. Do the operations in grouping symbols first (e.g., use rules 2-4 inside parentheses).
2. Calculate all the expressions with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

$$\begin{aligned}\frac{3^2+(6\bullet 2-1)}{5} &= \frac{3^2+(12-1)}{5}\\ &= \frac{3^2+(11)}{5}\\ &= \frac{9+(11)}{5} = \frac{20}{5} = 4\end{aligned}$$

The rules for order of operation often make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for \$1.50 each and 3 bags of peanuts for \$1.25 each. Write an expression for the total cost, and simplify the expression to find the total cost.

Expression: $\underbrace{2 \bullet (1.50)} + \underbrace{3 \bullet (1.25)}$

Simplification: $3.00 + 3.75 = 6.75$

In this problem it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

Note, however, that if we were to perform the operations in order from left to right (as we read the English language from left to right), we would obtain a different result:

$$2 \bullet 1.50 + 3 \bullet 1.25 = ((2 \bullet 1.50) + 3) \bullet 1.25 = (3 + 3) \bullet 1.25 = 6 \bullet 1.25 = 7.50.$$

MATHEMATICAL SYMBOLS AND LANGUAGE

add	$+$	subtract	$-$
multiply	$\times \quad \bullet$	divide	$\div \quad /$
is equal to	$=$	is not equal to	$\neq$
is greater than	$>$	is less than	$<$
is greater than or equal to	$\geq$	is less than or equal to	$\leq$
is approximately equal to	$\approx$	square root	$\sqrt{}$

Symbols for Multiplication

The product of 8 and 4 can be written as:

8 times 4 8×4 $8 \bullet 4$ (8)(4) $\begin{array}{r} 8 \\ \underline{\times\ 4} \end{array}$

In algebra, we generally avoid using the $\times$ for multiplication because it could be misinterpreted as the variable *x*, and we cautiously use the symbol $\bullet$ for multiplication because it could be misinterpreted as a decimal point. The product of 8 and the variable *x* is usually written simply as $8x$. This is sometimes called implied multiplication, because no symbol is used.

Symbols for Division

The quotient of 8 and 4 can be written as:

8 divided by 4 $8 \div 4$ $4\overline{\smash{)}8}$ $\frac{8}{4}$ 8/4

Fraction notation is generally the preferred notation in algebra.

Mathematical Separators

Parentheses "( )" and square brackets "[ ]" are used in mathematical language as separators. The expression inside the parentheses or brackets is to be considered as a single unit. Operations are performed inside the parentheses before the expression inside the parentheses is combined with anything outside the parentheses.

Example: $5(2 + 1)x = 5(3)x = 15x$.

The horizontal line used for a division problem is a separator. It separates the expressions above and below the line.

VARIABLES AND EXPRESSIONS

Uses of Variables

Example:	Type of mathematical sentence	Variables	How variables are used
$4 + 5x$	expression	x	x might be any number; if a value for x is given, the expression may be evaluated.
$12 = 2y + 2$	equation	y	y is an unknown that can be solved for; in this case, y must be 5.
$y = x + 10$	equation (function)	x and y	This equation shows a relation between two variables. As one variable changes, the other changes according to the prescription of the equation. For example: If $x = 1$, then $y = 11$. If $x = 5$, then $y = 15$.
$d = rt$	equation (formula)	d, r, and t	This equation is most commonly used as a formula to relate the quantities of **d**istance, **r**ate of speed, and **t**ime.

Using Shapes to Represent Variables

If the same shape (variable) is used more than once in an equation (or system of equations), it must represent the same value each place where it appears. Two different shapes (variables) in an equation (or system of equations) may represent the same values or different values.

This is allowed	This is allowed	This is not allowed
□ + □ = ○	□ + ⬯ = ◇	○ + ○ = △
7 + 7 = 14	6 + 6 = 12	6 + 4 = 10

Writing Expressions

The specialized system of notation and rules that is used for symbolic algebra is sometimes different than the notation used for arithmetic. For example:

- 54 means the sum of five tens and four ones, that is, $5(10) + 4$.
- $5\frac{1}{2}$ means the sum of five and one-half. That is, $5 + \frac{1}{2}$.
- $5x$ means the product of five and x, which can also be written $5(x)$ or $5 \bullet x$. We do not write $5 \times x$ because the multiplication symbol '$\times$' is easily confused with the variable x.

The Cups and Counters Model for Expressions and Equations

- This cup ▼ represents an unknown (or x).

  Draw the cup like this: **V**

- This upside-down cup ▲ represents the opposite of the unknown (or $-x$).

  Draw the upside down cup like this: **Λ**

- This counter $\boxed{+}$ represents 1 unit (or 1).

  Draw the counter like this: **+**

- This counter $\boxed{-}$ represents the opposite of 1 unit (or -1)

  Draw the counter like this: **–**

Zero pairs illustrate the <u>inverse property of addition</u>.

Here are some examples:

with counters	as a drawing	with cups	as a drawing
$\boxed{+}$ $\boxed{-}$ $\boxed{+}$ $\boxed{-}$ $\boxed{+}$ $\boxed{+}$ $\boxed{-}$ $\boxed{-}$	+ – + – + –	▼ ▼ ▲ ▲	V Λ
4 + (-4) = 0	-3 + 3 = 0	$2x + (-2x) = 0$	$-1x + 1x = 0$

Simplifying Expressions Using a Model

In mathematics, to <u>simplify</u> a numerical or algebraic expression is to convert the expression to a less complicated form.

We can illustrate simplifying expressions using a cups and counter model.

Example 1: Simplify $3(x + 2)$

Picture	Expression	What did you do?
V + + **V + +** **V + +**	$3(x + 2)$ $= 3x + 6$	Build the expression (think: 3 groups of $x + 2$) Simplify (the distributive property)

Example 2: Simplify $-(x - 3)$

Picture	Expression	What did you do?
V − − − ⇩ **Λ + + +**	$x - 3$ ⇩ $-(x - 3)$ $= -x + 3$	First build the expression inside the parentheses Then rebuild its opposite to remove parentheses. (the distributive property)

Example 3: Simplify $2x + 3(x - 4)$

Picture	Expression	What did you do?
V V V − − − − **V − − − −** **V − − − −**	$2x + 3(x - 4)$ $= 2x + 3x - 12$ $= 5x - 12$	Build the expression (think: $2x$ plus 3 groups of $x - 4$) Apply the distributive property (no action is taken with the model here); then collect like terms to simplify

Attend to Precision When Using the Words Negative and Opposite

A “right-side-up cup” represents an unknown value, such as *x*, and is represented by this picture: **V**

An “upside-down cup” represents the opposite of an unknown value, such as -*x* (read “the opposite of *x*), and is represented by this picture: **Λ**

Consider:

- If $x = 4$, then $-x = -4$.
- If $x = -5$, then $-x = -(-5) = 5$.
- If $x = 0$, then $-x = -0 = 0$.

In only the first example above is $-x$ a negative number.

Saying “negative *x*” improperly implies that $-x$ represents a negative number.

LINEAR EQUATIONS IN ONE VARIABLE

Summary of Properties Used for Solving Equations	
Properties of arithmetic govern the manipulation of expressions (mathematical phrases). These include:	
• Associative property of addition • Commutative property of addition • Additive identity property • Additive inverse property	• Associative property of multiplication • Commutative property of multiplication • Multiplicative identity property • Multiplicative inverse property
• Distributive property relating addition and multiplication	
Properties of equality govern the manipulation of equations (mathematical sentences). These include:	
• Addition property of equality Subtraction property of equality	• Multiplication property of equality Division property of equality

Solving Equations Using a Model

Let **+** represent 1

Let **–** represent -1

Let **V** represent the unknown

Let **Λ** represent the opposite of the unknown

Note that the following examples illustrate only one solution path. Others paths are possible to arrive at the same solutions.

Example 1: Solve $6 = 2(x + 1)$

Picture		Equation	What did you do?
+ + + + + +	V + V +	$6 = 2(x + 1)$ or $6 = 2x + 2$	Build the equation.
~~+~~ + + ~~+~~ + +	V ~~+~~ V ~~+~~	$6 = 2x + 2$ $\underline{-2 \qquad -2}$ $4 = 2x$	Take away 2 from both sides.
+ + → + + →	V V	$\frac{4}{2} = \frac{2x}{2}$ $2 = x$	Divide both sides by 2. Put units equally into cups.

Check: $6 = 2(2 + 1) = 2(3) = 6$

Solving Equations Using a Model (Continued)

Example 2: Solve $-3 = 3x + 6$

Picture	Equation	What did you do?
– – – ⋮ V V V + + + + + +	$-3 = 3x + 6$	Build the equation.
– – – ⋮ V V V – – – ⋮ + + + – – – ⋮ + + + – – – – – –	$-3 = 3x + 6$ $\underline{+(-6) \quad\quad +(-6)}$ $-9 = 3x$	Add -6 to each side. Remove zero pairs.
– – – → V – – – → V – – – → V	$\frac{-9}{3} = \frac{3x}{3}$ $-3 = x$	Divide both sides by 3. Put units equally into cups.

Check: $-3 = 3(-3) + 6 = -9 + 6 = -3$

Example 3: Solve $-2x – 1 = x – 4$

Picture	Equation	What did you do?
∧ ∧ ⋮ V – ⋮ – – – –	$-2x - 1 = x - 4$	Build the equation.
∧ ∧ ∧ ⋮ V ∧ – ⋮ – – – –	$-2x - 1 = x - 4$ $+(\underline{-x}) \quad +(\underline{-x})$ $-3x - 1 = \quad -4$	Add the opposite of x to both sides. Remove zero pairs
∧ ∧ ∧ ⋮ – – – $\cancel{–}$ $\cancel{–}$	$-3x - 1 = -4$ $\underline{-(-1)} \quad \underline{-(-1)}$ $-3x = -3$	Remove -1 from each side (This gives the same result as adding 1 to each side.)
V V V ⋮ + + +	$-3x = -3$ $(-1)(-3x) = (-1)(-3)$ $3x = 3$	Multiply both sides by -1. This has the effect of “taking the opposite of both sides.”
V → + V → + V → +	$\frac{3x}{3} = \frac{3}{3}$ $x = 1$	Divide both sides by 3. Put units equally into cups

Check: $-2(1) – 1 = 1 – 4 \rightarrow -2 – 1 = -3 \rightarrow -3 = -3$

Using Algebraic Techniques to Solve Equations

To solve equations using algebra:

- Use the properties of arithmetic to simplify each side of the equation (e.g. associative properties, commutative properties, inverse properties, distributive property).
- Use the properties of equality to isolate the variable (e.g. addition property of equality, multiplication property of equality).

Example: Solve: $3 - x + 3 = 5x - 2x - 2$ for x.

Equation	Comments
$3 - x + 3 = 5x - 2x - 2$ $6 - x = 3x - 2$	• collecting like terms: $5x - 2x = 3x$; note that this is an application of the distributive property, $(5 - 2)x = 3(x)$
$6 - x = 3x - 2$ $\underline{+2} \qquad \underline{+2}$ $8 - x = 3x$	• addition property of equality: add 2 to both sides • additive inverse property: $-2 + 2 = 0$
$8 - x = 3x$ $\underline{+x} \qquad \underline{+x}$ $8 = 4x$	• addition property of equality: add x to both sides • additive inverse property: $-x + x = 0$ • collect like terms $3x + x = (3+1)x = 4x$
$8 = 4x$ $\frac{8}{4} = \frac{4x}{4}$ $2 = x$	• multiplication property of equality: divide both sides by 4 (equivalent to multiplying both sides by $\frac{1}{4}$) • multiplicative identity property: $1x = x$

HEADS UP! Some equations have infinitely many solutions, and some equations have no solutions. All of the previous examples have exactly one solution. That is, there is just one value that makes the equation true.

$$\begin{aligned} 3x + 6 &= 3(x + 2) \\ 3x + 6 &= 3x + 6 \\ 3x &= 3x \\ x &= x \end{aligned}$$

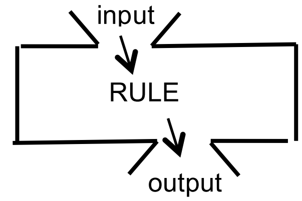
This statement is true for all x. Therefore there are infinitely many solutions.

$$\begin{aligned} 3x + 2 &= 3(x + 2) \\ 3x + 2 &= 3x + 6 \\ 2 &= 6 \end{aligned}$$

This statement is false, and therefore the equation has no solutions.

FUNCTIONS

A function is a rule that assigns to each input value a unique output value. Some ways to represent rules in mathematics are input-output tables, mapping diagrams, ordered pairs, equations, and graphs.



Examples that are Functions

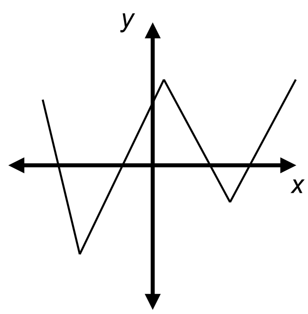
x input	y output
1	1
3	3
5	5
7	7
9	9

This table lists input values with unique output values.

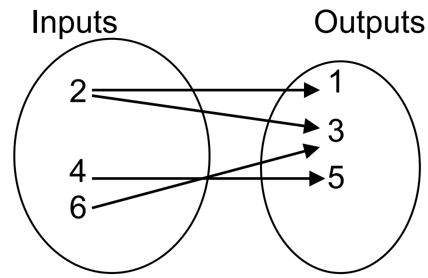
{(0, 2), (1, -2), (2, 2), (3,- 2)}

In this set of ordered pairs, each input value is assigned to a unique output value. Note that different input values may be assigned the same output value. In this example, both 1 and 3 are assigned the output value -2.

This graph represents a function because every vertical line through it intersects at most one point of the graph. In other words, each possible x-value corresponds to a unique y-value.



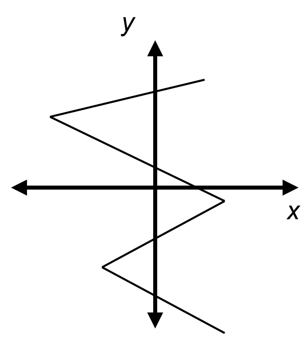
Examples that are NOT Functions



This mapping diagram is not a function. It is not permissible for the same input value (in this case 2) to be assigned two different output values.

Consider the set of pairs (x, y) that satisfy $x = y^2$, such as (0,0), (25, 5), and (25, -5). Since the input value, x = 25, corresponds to two different output values (y = 5 and y = -5), the -values are not a function of the x values.

This graph does not represent a function because some vertical lines (for example, the y-axis) intersect the graph in more than one point. In other words, some x-values correspond to more than one y-value.



LINEAR FUNCTIONS

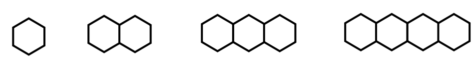
Using Multiple Representations to Describe Linear Functions (The Fourfold Way)

The fourfold way refers to a collection of four representations used to approach a math problem:

- Numbers (numerical approach, as by making a t-table)
- Pictures (visual approach, as with a picture or graph)
- Symbols (approaching the problem using algebraic symbols)
- Words (verbalizing a solution, orally or in writing)

Each approach may lead to a valid solution. Collectively they should lead to a complete and comprehensive solution, one that is readily accessible to more people and that provides more insight.

Example 1: Describe this toothpick pattern of hexagons using numbers, pictures, words, and symbols.



Numbers

Step #	number of toothpicks	Breaking apart numbers sometimes helps you see an explicit rule.
1	6	6 = 6 + (0)5
2	11	6 + 5 = 6 + (1)5
3	16	6 + 5 + 5 = 6 + (2)5
4	21	6 + 5 + 5 + 5 = 6 + (3)5
5	26	6 + 5 + 5 + 5 + 5 = 6 + (4)5
n	$5n + 1$	$5n + 1 = 6 + (n - 1)5$

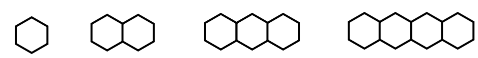
Words

One way to describe the hexagonal toothpick pattern is to start with 6 toothpicks and add 5 more toothpicks at each subsequent step. Notice that the number of 5's added at each step is equal to 1 less than the step number.

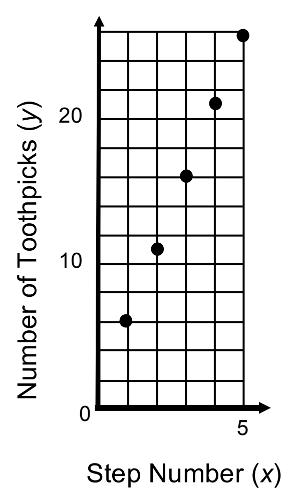
Symbols

A rule for finding the number of toothpicks at step n is $6 + (n - 1)5$, which can be simplified to $5n + 1$

Pictures



step 1 step 2 step 3 step 4



Note: we consider a graph to be a picture.

Using Multiple Representations to Describe Linear Functions (Continued)

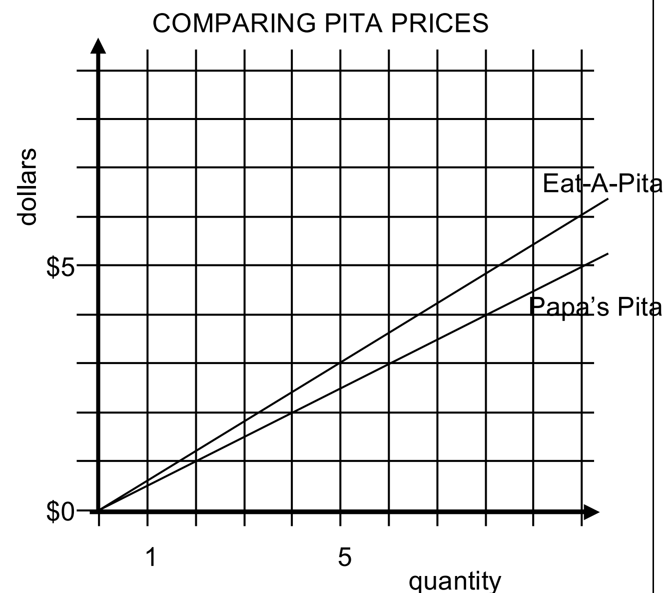
Example 2: At Papa's Pitas, 2 pitas cost \$1.00. At Eat-A-Pita, 5 pitas cost \$3.00. Assuming a proportional relationship between the number of pitas and their cost, use multiple representations to explore which store offers the better buy for pitas.

Numbers (make a table)

PAPA'S PITAS	
# of pitas (x)	cost (y)
2	\$1.00
4	\$2.00
6	\$3.00
8	\$4.00
10	\$5.00

EAT-A PITA	
# of pitas (x)	cost (y)
5	\$3.00
10	\$6.00
15	\$9.00
20	\$12.00
25	\$15.00

Pictures (make a graph)



Words (write sentences)

Based on the table, Papa's Pitas is the better buy.

At Papa's Pitas, you get 6 pitas for \$3.00. This means the unit price (cost for one pita) is \$0.50.

At Eat-A-Pita you only get 5 pitas for \$3.00. This means the unit price (cost for one pita) is \$0.60.

Symbols (write equations to relate the number of pitas to cost)

PAPA'S PITAS $y = 0.5x$

EAT-A PITA $y = 0.6x$

Notice that \$0.50 is the cost of one pita at Papa's Pita. This corresponds to the point (1, 0.5) on the graph.

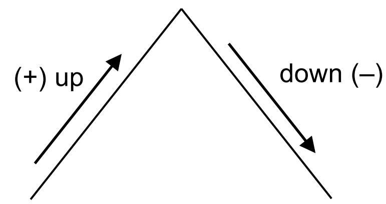
Notice that \$0.60 is the cost of one pita at Eat-A-Pita. This corresponds to the point (1, 0.6) on the graph.

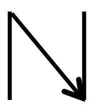
The equations above are both in the form $y = mx$. This is called a direct proportion equation because y is directly proportional to (is a constant multiple of) x. Graphs of direct proportion equations always go through the origin.

Slope

Roughly speaking, slope (m) is the "slant" of a line.

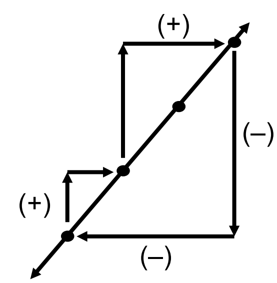
One way to think about whether slope is positive or negative is to imagine that the line is a portion of a mountain (m). Just as we read from left to right, we will move up and down the mountain from left to right. When moving up the mountain, the slope is positive. When moving down the mountain, the slope is negative. The steeper the mountain, the greater (in absolute value) the slope.





Another visual to help remember a negative slope is to think of the diagonal line segment in the middle of a capital N ("N for "negative").

To compute slope, draw a right triangle with directional arrows to make vertical and horizontal distances clear. Use directed line segments to show the vertical and horizontal changes**. First** move in a vertical direction and find the directional distance, and **then** move in a horizontal direction and find the directional distance.

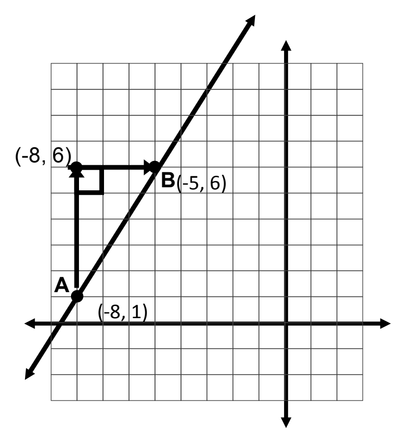


Slope (m) is computed as: $\dfrac{\text{vertical change}}{\text{horizontal change}}$ as you move from one point to another, or

$\dfrac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}}$ as you move from one point to another.

Example 1: If A (-8,1) and B (-5,6) are points on a line, then

$$m = \frac{\text{difference in } y}{\text{difference in } x} = \frac{6-1}{-5-(-8)} = \frac{5}{3}.$$



Example 2: If (a, b) and (c, d) are points on a line, then

$$m = \frac{\text{difference in } y}{\text{difference in } x} = \frac{d-b}{c-a}.$$

The formula in example 2 is the definition of the slope of a line.

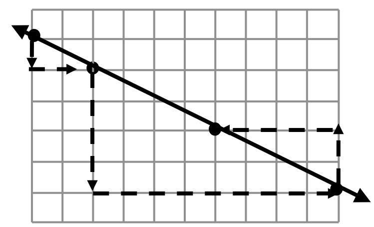
Some Strategies for Understanding Slope

A good understanding of the meaning of slope will better prepare you to make sense of linear functions. We emphasize the meaning of slope by

(1) finding slopes by "grid counting,"
(2) recognizing whether lines have positive or negative slopes, and
(3) using different points on a line to generate equivalent fractions for the slope.
(4) Further, we practice using the slope formula, given different kinds of information, and
(5) deepen understanding of slope by connecting it to concepts of geometry such as properties of parallel lines and similar triangles (in a future geometry lesson).

Example: For the line on the right, the slope is given by

$$-\frac{1}{2} = \frac{-1}{2} = \frac{2}{-4} = \frac{-4}{8}.$$



The Slope of Horizontal and Vertical Lines

<u>The slope of a horizontal line is zero</u>. A horizontal line is a line parallel to the *x*-axis. Every point on a horizontal line has the same *y*-coordinate, and the vertical change between any two positions on the line is zero. Hence

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{0}{\text{horizontal change}} = 0.$$

<u>The slope of a vertical line is undefined</u>**.** A vertical line is a line parallel to the *y*-axis. Every point on a vertical line has the same *x*-coordinate, and the horizontal change between any two points on the line is zero. Hence

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{vertical change}}{0}$$

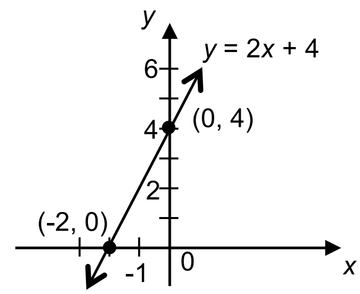
is undefined, since division by zero is undefined.

Intercepts

The <u>y-intercept</u> of a line is the y-coordinate of the point of intersection of the line and the y-axis. It is the y-value corresponding to $x = 0$.

The <u>x-intercept</u> of a line is the x-coordinate of the point of intersection of the line and the x-axis. It is the x-value corresponding to $y = 0$.

Example: For the line with equation $y = 2x + 4$, the y-intercept is obtained by solving the equation $y = 2(0) + 4$. Thus the y-intercept is 4. The point of intersection of the line and the y-axis is (0, 4).



Example: For the line with equation $y = 2x + 4$, the x-intercept is obtained by solving the equation $0 = 2x + 4$. Thus the x-intercept is -2. The point of intersection of the line and the x-axis is (-2, 0).

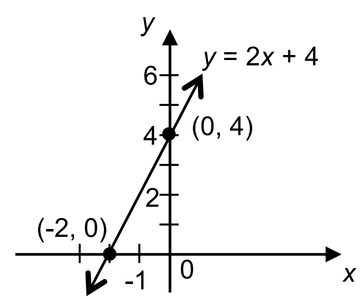
The Slope-Intercept Form of Linear Equations

<u>Slope-intercept form of a linear equation</u>: $y = mx + b$, where m = slope of the line and b = the y-intercept. This form of the equation is unique.

Example: Say you are given that the slope of a line is $\frac{-1}{3}$ and the y-intercept is -5. Then you know that the equation of the line is $y = \frac{-1}{3}x - 5$.

Example: For the line that passes through the points (0, 4) and (-2, 0), you are given the intercept, so find the slope.

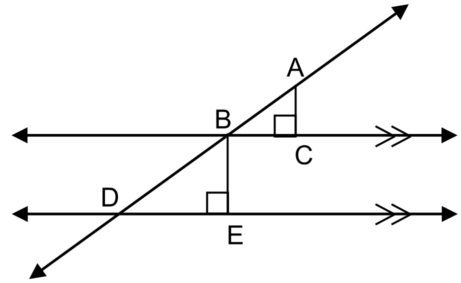
For example 2



$b = 4$ and $m = \dfrac{4-0}{0-(-2)} = 2$, so the equation of the line is $y = 2x + 4$.

Similar Triangles and Slope

Once the basic idea of slope is established, some properties of plane geometry help to further develop the concept. For example, we can establish that all triangles formed using a segment from a non-vertical line as the hypotenuse and vertical and horizontal segments as legs are similar to each other. Consider this diagram. Focus on $\triangle DEB$ and $\triangle BCA$.



$\angle BED \cong \angle ACB$	(Both are right angles, and all right angles are congruent to each other.)
$\angle BDE \cong \angle ABC$	(If parallel lines are cut by a transversal, then corresponding angles are congruent.)
$\angle DBE \cong \angle BAC$	(Sum of three angles in a triangle is 180°, and the other two angles have been determined.)

Since corresponding angles in the two triangles are congruent, $\triangle DEB \sim \triangle BCA$.

Once similarity is established, then the ratio of the corresponding sides will be the same, as this is a fundamental property of similarity. The slope ratio is one of these ratios. Specifically,

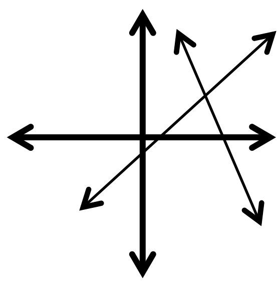
$$\frac{\text{directed length of vertical leg}}{\text{directed leg of horizontal leg}} = \text{slope of the line.}$$

You will learn more about the connections between similarity and slope as you study geometry.

SYSTEMS OF LINEAR EQUATIONS

A system of equations is a set of two or more equations in the same variables. A solution to the system of equations consists of values for the variables which, when substituted, make all equations simultaneously true.

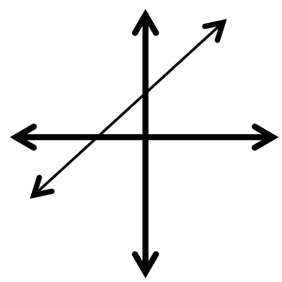
A system of linear equations has exactly one solution, infinitely many solutions, or no solution.



$$\begin{cases} y = x - 1 \\ y = -2x + 11 \end{cases}$$

One solution at (4, 3)

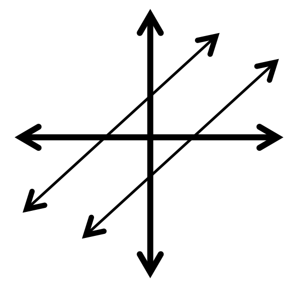
These two lines intersect in exactly one point.This is the only pair of x- and y-values that statisfies these two equations simultaneously.



$$\begin{cases} y = x - 1 \\ 3y = 3x - 3 \end{cases}$$

Infinitely many solutions

Since the equations are equivalent, the two lines coincide. Every point on the line represents a solution.



$$\begin{cases} y = 3x - 3 \\ y = 3x + 3 \end{cases}$$

No solution

Since these two lines are parallel, they do not intersect. Thus these two equations have no solution in common.

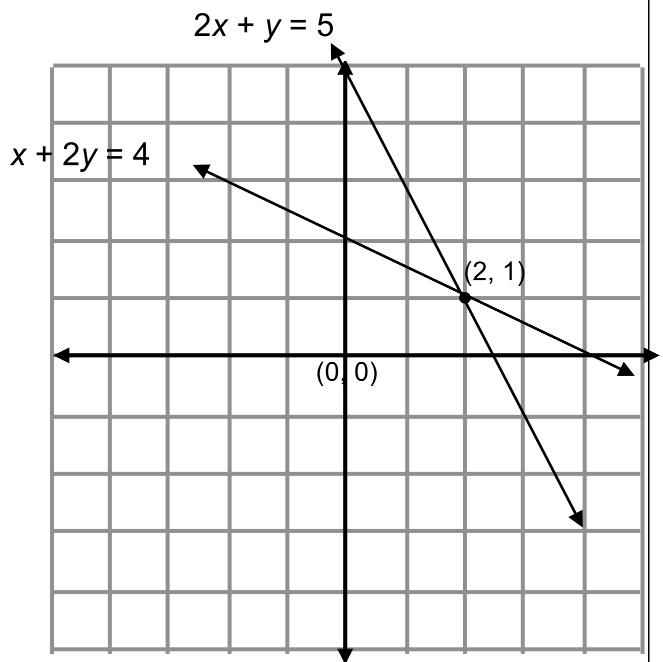
Solving a System of Linear Equations by Graphing

To solve a system of linear equations by graphing, graph both lines on the same set of axes and observe the point(s) of intersection, if any.

Example: Solve this system of equations by graphing:

$$\begin{cases} 2x + y = 5 \\ x + 2y = 4 \end{cases}$$

by graphing.



1. Change each equation to slope-intercept form.

$2x + y = 5 \longrightarrow y = -2x + 5$

$x + 2y = 4 \longrightarrow y = -\frac{1}{2}x + 2$

2. Graph each linear equation.

3. Observe the intersection of the lines, (2, 1). This represents the solution to the system. In other words, these are the *x*- and *y*- values that satisfy **both** equations.

   Remember that not every system of equations has exactly one solutuion.

4. Check by substituting solutions in the original equations to be sure they are correct.

$2x + y = 5 \longrightarrow 2(2) + 1 = 5$ (true)

$x + 2y = 4 \longrightarrow 2 + 2(1) = 4$ (true)

Solving a System of Linear Equations by Substitution

Example: Solve this system of linear equations by substitution.

$$\begin{cases} 2x + y = 5 \\ x + 2y = 4 \end{cases}$$

1. In at least one of the equations, solve for one variable in terms of the other. For example,

$$2x + y = 5 \longrightarrow y = -2x + 5$$

2. Use the substitution property to replace the expression equivalent to y in the second equation and solve for x.

$$x + 2y = 4 \longrightarrow x + 2(\underbrace{-2x + 5}_{y \text{ from step 1}}) = 4$$

$$x - 4x + 10 = 4$$

$$-3x = -6$$

$$x = 2$$

3. Substitute x into one of the original equations to find y.

$$2x + y = 5 \longrightarrow 2(\underbrace{2}_{x \text{ from step 2}}) + y = 5$$

$$y = 1$$

4. Substitute x and y into the other original equation to check.

$$x + 2y = 4 \longrightarrow \underbrace{2}_{x \text{ from step 2}} + 2(\underbrace{1}_{y \text{ from step 3}}) = 4 \text{ (true)}$$

Solving a System of Linear Equations by Elimination

Elimination, which applies the multiplication property of equality and the addition property of equality, is another method for solving a system of linear equations. Here is an example.

Example: Solve this system of linear equations by elimination.

$$\begin{cases} 2x + y = 5 & [1] \\ x + 2y = 4 & [2] \end{cases}$$

We will number each equation with brackets to help keep track of them.

1. Use the multiplication property of equality. Multiply both sides of one (or both) equations by some number that will make one of the variable expressions in each equation opposites of each other. In this case, we might multiply both sides of the first equation by -2.

$$-2(2x + y) = -2(5) \longrightarrow -4x - 2y = -10 \quad [3]$$

2. Use the addition property of equality. Add expressions on each side of the equation together. Solve the new equation.

$$\begin{aligned} -4x - 2y &= -10 \quad [3] \\ \underline{x + 2y} &= \underline{4 \qquad\; [2]} \\ -3x &= -6 \\ x &= 2 \end{aligned}$$

3. Substitute into one of the original equations to find y.

$$[1] \quad 2x + y = 5 \longrightarrow 2(2) + y = 5$$
$$y = 1$$

4. Substitute into the other original equation to check.

$$[2] \quad x + 2y = 4 \longrightarrow 2 + 2(1) = 4 \quad \text{(true)}$$

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